Modeling the Mobile Oil Recovery Problem as a Multiobjective Vehicle Routing Problem

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Abstract. The Mobile Oil Recovery (MOR) unit is a truck able to pump marginal wells in a petrol field. The goal of the MOR optimization Problem (MORP) is to optimize both the oil extraction and the travel costs. We describe several formulations for the MORP using a single vehicle or a fleet of vehicles. We have also strengthened them by improving the subtour elimination constraints. Optimality is proved for instances close to reality with up to 200 nodes.

Keywords: Vehicle routing problem, prize-collecting, multiobjective.

1 Introduction

Much effort has been made to increase the oil production in Brazil though the use of new technologies. As a consequence, the Brazilian oil production has met the country's need in 2006 and the country is globally self sufficient. The Rio Grande do Norte basin has been exploited for the last 30 years and about 98% of the oil wells are pumped using artificial lift systems. One such system is the Mobile Oil Recovery (MOR) unit. It consists of a truck equipped with a pumping system. The unit starts its tour at the depot, then it pumps several wells before returning to the depot at the end of the day. Whenever the unit's tank is full, an auxiliary vehicle is used to transfer the oil from the MOR unit to its own tank and to carry it to the depot. Thus, the MOR unit capacity can be considered unlimited.

The MOR optimization Problem (MORP) is a multiobjective problem which consists in finding a set of wells to be pumped in a working day to maximize the oil extraction and to minimize the travel time. The two objectives are opposite, one pushing to increase profit and the other to reduce costs. With one MOR unit, the problem is close to the Selective Traveling Salesman Problem which is also called Orienteering Problem or Maximum Collection Problem [8]. With a fleet of vehicles, the problem becomes a Vehicle Routing Problem (VRP) close to the Prize-Collecting VRP [2]. For further investigation on routing problems, readers are referred to the following works: the state of the art on exact and

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approximated methods for the VRP and its variants are found in [14] and an overview covering about 500 papers on classical routing problems are found in [9]. For multiobjective solutions strategies on routing problems, see [3,7].

A mathematical formulation for the MORP is proposed in [13] for a single MOR unit. Heuristics applications of the MORP are presented in [1,13]. We propose several formulations for the MORP with a single vehicle or a fleet of vehicles. They are strengthened by improving the subtour elimination constraints. Instances with up to 200 nodes, close to reality, are solved.

The paper is organized as follows: the problem definition and formulations for one unit are presented in Section 2. Sections 3 and 4 are devoted to the MORP with several units. Computational results are shown in Section 5 and final remarks are made in Section 6.

2 Formulations Using One Vehicle

The geographical data (roads, wells and depot) are modeled as an undirected graph $G = (N, E)$. G is preprocessed to build a complete digraph $G' = (V, A)$ where V is the set of wells and the depot v_0. Let d_{ij} be the shortest distance from i to j, $\forall(i, j) \in A$, and let s be the MOR unit average speed. Thus, for every arc of G', the travel time t_{ij} is computed as $t_{ij} = d_{ij}/s$.

Let t'_i be the total operation time at well i (time to connect the unit, to pump, and to disconnect the unit) and let p_i be its oil production. Let P and T be respectively the total oil production and the total working time of a MOR unit. Moreover, $\overline{T}$ is the maximal time an MOR unit can work in a day. Wells can be exploited only once a day as in the previous works [1,13].

Given K, the total number of MOR units, the MORP consists in defining one circuit $\tau = \{v_0, v_{\sigma 1}, v_{\sigma 2}, ..., v_{\sigma k}, v_0\}$ for each MOR unit, where σ is the position of wells in the circuit to be exploited in a day, such that P is maximized and T is minimized. The time limit $T \leq \overline{T}$ has to be satisfied.

As far as we know, only one formulation has been proposed in the literature for the MORP [13]. It considers one vehicle and the optimization is done in two phases: first, the maximal amount of oil is computed, and second, the shortest route to extract this amount is computed. In this section, our contributions improve the formulation proposed in [13] as follows: (i) remove the constraint ensuring the MOR unit returns to the depot because it is redundant, (ii) simplify the flow conservation constraints, (iii) test different strategies to eliminate invalid subtours, and (iv) strengthen the subtour elimination constraints.

Let $f_{ij} \in \{0, 1\}$ be the decision variable on the choice of arc (i, j) and let x_i be the binary variables which specify if well i is exploited or not. The first optimization phase for the MORP is given as follows:

$$\max \; P = \sum_{i \in V \setminus \{v_0\}} p_i \cdot x_i \qquad s.t. \tag{1}$$

$$\sum_{i \in V \setminus \{v_0\}} t'_i \cdot x_i + \sum_{(i,j) \in A} t_{ij} \cdot f_{ij} \leq \overline{T} \tag{2}$$

$$\sum_{j:(j,i)\in A} f_{ji} - \sum_{j:(i,j)\in A} f_{ij} = 0 \quad \forall i \in V\backslash\{v_0\} \tag{3}$$

$$\sum_{j:(j,i)\in A} f_{ji} = x_i \quad \forall i \in V\backslash\{v_0\} \tag{4}$$

$$\sum_{j\in V} f_{0j} = 1 \tag{5}$$

$$\text{(subtour eliminations constraints)} \tag{6}$$

$$x_i \in \{0,1\} \qquad \forall i \in V \tag{7}$$

$$f_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \tag{8}$$

The objective function (1) aims at minimizing the oil extraction. Inequality (2) limits the working time (travel and operation time) to $\overline{T}$. The flow conservation constraints are (3) and (4). Restriction (5) guarantees the tour starts at the depot. Variables x_i and f_{ij} are respectively defined in Constraints (7) and (8). We discuss in Section 2.1 the use of several subtour elimination constraints: those of Miller and Tucker and Zemlin (MTZ) [5,11], and those of Gavish and Graves using either aggregated (GGA) or disaggregated flow (GGD) [6]. GGA constraints are used in [13].

The objective of the second optimization phase is to minimize the working time (9) subject to Constraints (3)–(8) and (10). Constraint (10) restricts the total production to be equal to the total optimal prize P^* obtained in the first phase.

$$\min \; T = \sum_{i\in V\backslash\{v_0\}} t'_i \cdot x_i + \sum_{(i,j)\in A} t_{ij} \cdot f_{ij} \qquad s.t. \tag{9}$$

$$\sum_{i\in V\backslash\{v_0\}} p_i \cdot x_i = P^* \tag{10}$$

Constraints (3)–(8).

2.1 Subtour Eliminations Constraints

A subtour is defined by any ordered subset of vertices. For the MORP, only subtours starting and ending at the depot v_0 are valid. Subtour constraints have been evaluated in the literature for the TSP problem, see e.g. [15]. MTZ, GGA and GGD subtour elimination constraints for the MORP, and some improvements are described below.

An upper bound on the number M of wells that can be exploited in a working day can be computed. Considering the working time of the MOR unit, a simple procedure consists in computing M by sorting the wells in increasing order of operation time t'_i [13]. Thus, M is such that:

$$\sum_{i=1}^{M} t'_i \leq \overline{T} \leq \sum_{i=1}^{M+1} t'_i. \tag{11}$$

We propose to strengthen the value of M by using also the minimum travel time to arrive at each node. Moreover, the vehicle must return to the depot and the minimal time to return to the depot is also considered. M is given as:

$$\sum_{i=1}^{M} \left(t'_i + \underset{j \in V}{min} \{t_{ji}\} \right) \leq \overline{T} - \underset{j \in V}{min} \{t_{j0}\} \leq \sum_{i=1}^{M+1} \left(t'_i + \underset{j \in V}{min} \{t_{ji}\} \right) \tag{12}$$

Lifted Miller and Tucker and Zemlin Constraints. The Miller, Tucker and Zemlin constraints define a topological order to eliminate invalid subtours. Variables u_i state the order well i appears in the tour. However, for the MORP, the depot appears twice (at the beginning and at the end). Thus, one can duplicate the depot and work on a support graph. We consider instead the depot only at the beginning of the topological design. This can be done since the flow structure defined by variables f_{ij} and x_i, and Constraints (3)–(5) guarantees the return to the depot. The corresponding MTZ constraints for the MORP is given in Equations (13)–(14).

$$u_i - u_j + M \cdot f_{ij} \leq M - 1 \qquad \forall (i,j) \in A, j \neq \{v_0\} \tag{13}$$

$$0 \leq u_i \leq M \qquad \forall i \in V \tag{14}$$

There is $O(|V|^2)$ of such constraints, improved by M. They can be lifted using the same ideas as Desrochers and Laporte [5]. It consists in adding a valid non-negative term $\alpha_{ji} f_{ji}$ to the Inequalities (13): $u_i - u_j + M \cdot f_{ij} + \alpha_{ji} \cdot f_{ji} \leq M - 1$. If $f_{ji} = 0$, then α_{ji} may take any value. Suppose now $f_{ji} = 1$. Then, the MOR unit exploits well $j \neq v_0$ before well i, $u_i = u_j + 1$. Thus, $f_{ji} = 1$ implies $f_{ij} = 0$ due to Constraints (3) and (4). By substitution, we obtain $\alpha_{ji} \leq M - 2$. The larger α_{ji}, the stronger is the lift. Thus, $\alpha_{ji} = M - 2$. A lifted version of Constraints (13) is given in Inequalities (15).

$$u_i - u_j + M \cdot f_{ij} + (M-2) \cdot f_{ji} \leq M - 1 \quad \forall (i,j) \in A, j \neq v_0 \tag{15}$$

Gavish and Graves Constraints. The Gavish and Graves [6] approach removes invalid subtours by building a network flow. A flow is sent to the nodes of the tour. Each node consumes one unit. In disaggregated flow, a specific flow is sent from the source to each node [4,10]. Otherwise, if the flow is not specified, it is an aggregated flow.

Let y_{ij} be the flow variable on arc (i,j). Thus, GGA constraints for the MORP are given in Equations (16)–(18). Constraints (16) are the flow conservation constraints. Inequalities (17) state a flow uses the arc (i, j) if it is selected. These constraints are strengthened by using M. In this strategy, there are $O(|V|^2)$ constraints and variables.

$$\sum_{j:(j,i)\in A} y_{ji} - \sum_{j:(i,j)\in A} y_{ij} = x_i \quad \forall i \in V\backslash\{v_0\} \tag{16}$$

$$y_{ij} \leq M \cdot f_{ij} \qquad \forall (i,j) \in A, \tag{17}$$

$$y_{ij} \geq 0 \qquad \forall (i,j) \in A \tag{18}$$

The GGD version is given in Constraints (19)–(22). Let y_{ij}^k be the variable specifying if flow for node k traverses arc (i, j) or not. Constraints (19) are the flow conservation constraints. Equations (20) state that a flow unit is sent from the source to each node k. Restrictions (21) specify that flow for node k traverses arc (i, j) if and only if it is used. This strategy implies $O(|V|^3)$ constraints and variables. Usually, it produces a better linear relaxation than using the aggregated flow.

$$\sum_{j:(i,j)\in A} y_{ij}^k - \sum_{j:(j,i)\in A} y_{ji}^k = 0 \qquad \forall k \in V\backslash\{v_0\}, \forall i \in V\backslash\{v_0, k\} \tag{19}$$

$$\sum_{j:(0,j)\in A} y_{0j}^k = x_k \qquad \forall k \in V\backslash\{v_0\} \tag{20}$$

$$y_{ij}^k \leq f_{ij} \qquad \forall k \in V\backslash\{v_0\}, \forall (i,j) \in A \tag{21}$$

$$y_{ij}^k \geq 0 \qquad \forall k \in V\backslash\{v_0\}, \forall (i,j) \in A \tag{22}$$

3 A Three-Indexed Formulation Using Several Vehicles

Let x_i^k be a decision variable that specifies if well i is exploited by the vehicle k or not. Variables $f_{ij}^k \in \{0, 1\}$ state if vehicle k exploits well j after well i or not. $P(K)$ is the total profit collected using the K MOR units. All other terms are defined in Section 2. The three-indexed formulation is as follows:

$$\max \ P(K) = \sum_{i\in V\backslash\{v_0\}} p_i \cdot \sum_{k=1}^{K} x_i^k \qquad s.t. \tag{23}$$

$$\sum_{i\in V\backslash\{v_0\}} t_i' \cdot x_i^k + \sum_{(i,j)\in A} t_{ij} \cdot f_{ij}^k \leq \overline{T} \qquad \forall k = 1, ..., K \tag{24}$$

$$\sum_{j:(j,i)\in A} f_{ji}^k - \sum_{j:(i,j)\in A} f_{ij}^k = 0 \qquad \forall k = 1, ..., K, \forall i \in V\backslash\{v_0, k\} \tag{25}$$

$$\sum_{j:(0,j)\in A} f_{0j}^k \leq 1 \qquad \forall k = 1, ..., K \tag{26}$$

$$\sum_{j:(j,i)\in A} f_{ji}^k = x_i^k \qquad \forall k = 1, ..., K,\ \forall i \in V\backslash\{v_0\} \tag{27}$$

$$\sum_{k=1}^{K} x_i^k \leq 1 \qquad \forall i \in V\backslash\{v_0\} \tag{28}$$

$$\sum_{j:(j,i)\in A} y_{ji} - \sum_{j:(i,j)\in A} y_{ij} = \sum_{k=1}^{K} x_i^k \qquad \forall i \in V\backslash\{v_0\} \tag{29}$$

$$y_{ij} \leq M \cdot \sum_{k=1}^{K} f_{ij}^k \qquad \forall (i,j) \in A, j \neq v_0 \tag{30}$$

$$y_{ij} \geq 0 \qquad \forall (i,j) \in A \tag{31}$$

$$x_i^k \in \{0,1\} \qquad \forall k = 1, ..., K, \forall i \in V\backslash\{v_0\} \tag{32}$$

$$f_{ij}^k \in \{0,1\} \qquad \forall k = 1, ..., K, \forall (i,j) \in A \tag{33}$$

Restrictions (24) limit the units work in a day. The flow conservation constraints are defined in (25) and (26). Constraints (27) ensure that unit k pass though an arc (i,j) only if it exploits well j. Inequalities (28) specify that at most one unit visits a well in a day. Constraints (29) and (30) are the GGA subtour elimination constraints. Constraints (31)–(33) are the variables definition. This formulation contains $O(|V^3|)$ variables and constraints. The GGA constraints are chosen according to the computational results for one vehicle (Section 5). Obviously, other strategies could be used as well.

4 A Two-Indexed Formulation Using Several Vehicles

We do not explicitly define which unit exploits well i as every unit has the same characteristics (homogeneous fleet). A similar idea was previously used, for example, in [12]. Variables f_{ij} and x_i are defined in Section 2. Additionally, variables d_i specify the date (time) well i is visited by a vehicle in a day. The two-indexed formulation is given as follows:

$$\max\ P = \sum_{i\in V\backslash\{v_0\}} p_i \cdot x_i \qquad s.t. \tag{34}$$

$$\sum_{j:(j,i)\in A} f_{ji} - \sum_{j:(i,j)\in A} f_{ij} = 0 \qquad \forall i \in V\backslash\{v_0\} \tag{35}$$

$$\sum_{j:(j,i)\in A} f_{ji} = x_i \qquad \forall i \in V\backslash\{v_0\} \tag{36}$$

$$\sum_{j\in V} f_{0j} = K \tag{37}$$

$$d_i - d_j + (\overline{T} + t'_i + t_{ij}) \cdot f_{ij} + (\overline{T} - t'_j - t_{ji}) \cdot f_{ji} \leq \overline{T} \qquad \forall (i,j) \in A, i,j \neq v_0 \tag{38}$$

$$d_i \geq t_{0i} \cdot f_{0i} + \sum_{j\neq v_0} (t_{0j} + t'_j + t_{ji}) \cdot f_{ji} \qquad \forall i \in V\backslash\{v_0\} \tag{39}$$

$$d_i \leq \overline{T} - (t'_i + t_{i0}) \cdot f_{i0} - \sum_{j\neq v_0} (t'_i + t_{ij} + t'_j + t_{j0}) \cdot f_{ij} \qquad \forall i \in V\backslash\{v_0\} \tag{40}$$

$$x_i \in \{0,1\} \qquad \forall i \in V \tag{41}$$

$$f_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \tag{42}$$

$$d_i \geq 0 \qquad \forall i \in V\backslash\{v_0\} \tag{43}$$

The flow conservation is given in Constraints (35). Restrictions (36) ensure arc (i,j) is used if well j is exploited. Inequalities (37) state K MOR units are used. Constraints (38) link the time node j is visited, to the time node i is visited, and to the selection of arc (i,j). This is an adaptation of the lifted MTZ constraints (see Section 2.1). Inequalities (39) and (40) define generalized lower and upper bounds on the time node i is visited. Inequalities (39) link the time node i is visited to variables f_{ji}. At most one of the arcs entering node i is used. Thus, d_i is at least equal to the minimal time required to arrive at node i, either by going from v_0 to i or by going from j to i. The same idea applies to the Inequalities (40). Variables x_i^k, f_{ij}^k and d_i are defined respectively by Constraints (41) to (43). The two-indexed formulation has $O(|V|^2)$ variables and constraints. MTZ is used since the time constraints definition is straightforward and the formulation has still $O(|V|^2)$ variables.

5 Computational Results

The computational experiments were carried out on an Intel Core 2 Duo with 2.66 GHz clock and 4Gb of RAM memory, using CPLEX 11 under default parameters. Instances were generated using a geographical information system to simulate real situations. Comparison among the proposed formulations are measured in terms of time to prove optimality and of linear relaxation.

In the Tables 1 and 2, each line corresponds to an instance. For each instance, the working day length (L) in minutes, the number of wells ($|V|$) and its optimal production (P^*) are given. For each formulation, the linear relaxation value (RL*), the time (T) spent by the unit in the optimal solution, the time (time(s)) required to prove optimality in seconds (rounded up), and the number of nodes (nodes) explored in the branch-and-bound tree are presented. The symbol ($-$) means the solver did not prove optimality because it ran out of memory. When the optimal solution is unknown, the best integer solution found so far is identified by "($\geq$ value)".

Table 1 summarizes the results for the formulations for one vehicle using the MTZ, GGA or GGD subtour elimination constraints. From the computational results, GGA proves optimality faster than MTZ and GGD for 9 instances. MTZ proves optimality faster than GGA and GGD for 7 instances. In spite of having the worst linear relaxation, MTZ is able to prove optimality for instances with up to 200 nodes. GGD consumes a lot of time even if it produces good linear relaxation. An interesting result on the linear relaxation is found for $L = 480$ and $|V| = 20$: the GGA linear relaxation is better than the linear relaxation of GGD. This happens here because the value of M is equal to the optimal amount of wells exploited in a day.

We have also run the second optimization phase for all instances presented in Table 1. The time T was only improved for the instance with $L = 480$ and $|V| = 120$ ($T^* = 479.5$ instead of $T = 480$). Thus, results of the second optimization phase were not tested for several vehicles. Even so, it remains valuable since it takes place in the global decision process of the problem.

The results for the two-indexed and the three-indexed formulations are presented in Table 2. The number of vehicles used (K) and the sum of the total time spent by all the MOR units (T') are given. The three-indexed formulation produces a better linear relaxation than the two-indexed formulation. However,

Table 1. The first optimization phase for the MORP using one vehicle

			MTZ				GGA				GGD			
L	$\|V\|$	P*	RL*	T	time	nodes	RL*	T	time	nodes	RL*	T	time	nodes
480	20	12.60	17.21	477	2	1292	12.92	477	0	1	13.41	477	11	17
480	30	15.88	18.38	477	5	2337	17.19	477	6	979	16.61	477	841	191
480	40	15.88	18.38	477	7	2081	17.19	477	6	963	16.61	477	30137	255
480	60	15.84	18.43	467	15	3203	16.86	467	3	90	-	-	-	-
480	80	9.97	13.80	479	83	5429	11.67	479	32	1410	-	-	-	-
480	120	18.73	19.09	480	73	2996	19.05	480	204	1910	-	-	-	-
480	160	19.10	19.47	480	240	2095	19.46	480	2540	3378	-	-	-	-
480	200	19.64	19.82	480	62	3256	19.77	480	20626	2360	-	-	-	-
960	20	24.45	32.11	952	817	915570	28.39	952	98	14807	25.65	960	452	264
960	30	31.65	35.93	950	424	228949	35.29	950	446	46898	32.43	950	5313	527
960	40	19.76	24.17	909	406	40244	23.84	909	135	10898	22.34	909	63631	2401
960	60	31.65	35.96	950	974	301668	35.18	950	377	33257	32.26	-	-	-
960	80	37.70	38.05	959.5	3240	57320	38.00	959.5	866	21595	-	-	-	-
960	120	37.99	38.41	960	2764	36803	38.42	960	872	6716	-	-	-	-
960	160	40.05	40.19	960	377	5893	40.16	960	585	877	-	-	-	-
960	200	40.05	40.19	960	420	6672	40.16	960	789	951	-	-	-	-

Table 2. The first optimization phase for the MORP using several vehicles

				Two-indexed formulation				Three-indexed formulation			
K	L	$\|V\|$	P^*	RL*	T'	time (s)	nodes	RL*	T'	time (s)	nodes
2	480	10	20.97	27.74	911	2	8416	24.09	**870**	22	9139
2	480	20	24.45	29.55	953	74	41047	25.08	953	4	487
2	480	30	31.16	35.79	941	51	51661	33.78	941	1149	75216
2	480	40	31.16	35.79	941	646	84815	33.78	941	1315	64397
2	480	50	28.99	34.70	928	654	94922	30.93	928	7929	101935
2	480	60	30.39	35.78	946	326	57405	32.99	946	1619	60724
2	480	70	25.88	32.86	916	2	3374	30.43	916	1219	26108
2	480	80	19.35	27.03	881	78	69927	22.86	**876**	16426	313421
2	960	20	$\geq$ 46.51	55.89	-	-	-	52.75	-	-	-
2	960	30	$\geq$ 62.26	69.44	-	-	-	68.57	-	-	-
3	480	10	29.82	29.82	909	1	503	29.82	**901**	2	639
3	480	20	33.72	40.42	943	553	704266	36.73	943	27788	214856
3	480	30	45.49	52.31	943	2394	2269664	49.86	-	-	-
3	960	20	62.16	62.16	933	134	138791	62.16	-	-	-
3	960	30	$\geq$ 88.78	98.44	-	-	-	97.63	-	-	-

the two-indexed formulation performs better to compute the optimal solution. In addition to the number of wells, the problem becomes more difficult when the number of vehicles increases. Moreover, the working day limit also contributes to the difficulty of the problem. Results suggest it is suitable to use a small time window (480 minutes). The three-indexed formulation found sometimes a smaller value of T' as shown in bold.

6 Concluding Remarks

Several formulations for the MORP are proposed in this work and the first ever results using several vehicles are presented. Additionally, we proposed to improve the subtour constraints by taking advantage of the time window. Thus, instances close to reality (up to 200 wells) are solved. Among the formulations for one vehicle, GGA performs globally better than MTZ and GGD to prove optimality. For several vehicles, the two-indexed formulation is faster to prove optimality in spite of weaker linear relaxations.

Computational experiments show that the time window restriction plays a key role in computing an optimal solution: the smaller the time window, the easier the problem to solve. Optimal solutions can be computed for medium-sized instances with two MOR units. When using three vehicles, this does not hold as the CPU time increases dramatically for small instances.

The larger instances used here are larger than the problems considered by the company in the Rio Grande do Norte Basin. Consequently, the oil company is now able to compute the optimal solution for the MORP instead of using solutions given by heuristics. It could be interesting in future work to investigate instances characteristics to specify situations where the second optimization phase becomes really useful. Moreover, for large time windows, we could investigate an approach to split it.

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