Classification of ambiguities in a visual language to query geographical databases

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Abstract:

Geographic Information Systems (GIS) manage geographical data and present the results visually using maps. Visual languages are well adapted to query such data. We propose to express queries sent to a GIS using symbolic maps with metaphors, i.e. visual representation of the spatial relationships making up the query.

Visual languages suffer from the appearance of ambiguity. We distinguish visual ambiguities from selection ambiguities. Visual ambiguities appear when a given visual representation of a query corresponds with several interpretations. In order to define new spatial relationship, the user points out one (or several) metaphor(s) already available in the restitution space. Selection ambiguities appear when a given selection corresponds with several metaphors.

We suggest palliating visual and selection ambiguities by associating a placing method with composition automata. The placing method insures to minimize level of ambiguity. We determine levels of ambiguity and user interaction complexity depending on the required expressive power. The higher the desired expressive power is, the higher the level of ambiguity is and thus the more complex the user interaction is. A prototype has been implemented to validate the placing method and the automaton allowing the highest expressive power.

Keywords: Geographic Information Systems (GIS), Visual Query Languages, Management of Ambiguities
1. Introduction

A lot of efforts are under progress to elaborate innovative solutions for the introduction of Geographic Information Systems (GIS) in the common life (e.g., urban communities, companies, a basic tool of an operating system) [3, 20, 26]. GIS rely on a database to manipulate traditional alphanumeric data and a spatial representation of a phenomenon. Languages to query such a database are based on an extension of the relational algebra to query the alphanumeric part. The problem is therefore to introduce the manipulations of spatial concepts. The main propositions extend SQL [2], either by adding new clauses [19, 30, 33] or new operators are defined [21, 25].

To take into account the visual capabilities of a human being (quick integration of a large set of data, spatial analysis), a visual representation of a visual phenomenon may be a promising approach to query a database for geographical applications. The basic paradigm is the metaphor. This concept helps the user to elaborate a mental model of a fuzzy notion with a well-known notion [6, 11 and 15]. A particular research domain is the integration of the visual dimension in a query language [1, 7, 9, 10, 12, 13, 14, 23, 24, 31, 34, 35 and 36]. A database language may provide two main activities: the data definition and the data manipulation. We focus here on the second part.

Visual languages have an important drawback: the ambiguities during the man-machine communication process. We propose in this article to tackle this problem. The aim of this article is to study the ambiguities due to the visual representation of spatial relationships while an end-user defines a query on a geographical database with a visual language.

The visual representation of a spatial query must take into account both the specificities of spatial queries and visual languages. A spatial query is the application of conventional database operators (e.g., alphanumeric selection) and spatial operators (e.g., intersection, adjacency). To provide a relevant expressive power, a query language must offer the mix of these two kinds of operators. Whatever the followed approach (e.g., textual, visual), the query language must provide operators closed on the geographic domain. A geographical data has at least two components (an alphanumeric part and a spatial part); the result of a spatial relationship (e.g., an intersection) must be a geographical data with at least two parts (an alphanumeric part and a spatial part). To provide such a capability, we exclude the
languages based on a predicate approach. This constraint is much more important as soon as the applications deal with network components (e.g., the important aspect is not to know if a path exists or not between two towns but the path itself). The closeness of the spatial operators allows the combination of spatial operators (and therefore a higher expressive power). To simplify the presentation, we use in this article some examples based on two well-known spatial operators (i.e., the intersection and the adjacency).\footnote{Even if we can’t be exhaustive, this work can easily be extended to other topological operators as those defined in [16]. Direction (e.g., east, south) or metrics operators may need a special study.}

The benefits of visual languages are now well accepted [4, 6, 8, 9, 32]: quick acquisition of a wide quantity of information, deduction of information by the mean of spatial analysis, syntax errors free languages. To really facilitate the interaction with an end-user, visual languages must provide two key properties: to be declarative and to adopt a « query-by-example » (QBE) philosophy [37, 38]. A language is declarative whenever the user defines the properties data must verify but not the way to obtain them. This opportunity frees the user from the elaboration of an execution plan. He or she can concentrate on the semantic of the application instead of the computer-language problems. A query language with a QBE philosophy allows the user expressing a query by providing an example of the result. This technique frees the user from the knowledge of the database schema, the associated operators and the syntax to define a query.

The common adopted architecture for a visual language is based on three levels (Figure 1). This architecture allows independence between the internal level of a database and the metaphor provided to an end-user as parts of a query language. In this article, we are concerned with the Interface level.

Figure 1 – Architecture of a data manipulation language with a visual philosophy

The definition of visual representations constitutes the vocabulary of a visual language. This vocabulary must be relevant for an end-user. The two main visual
components to favor are the icon, as defined in [9] (for a conventional database) and a map (for a geographical database). A map may be an image of the reality (e.g., a reactive map [22]) or a symbolic map (e.g., the spatial relationships defined by the end-user are independent of the extensions stored in the database).

Visual languages dealing with symbolic maps can be divided into two main families: the languages with a drawing tool [17, 28 and 29] and the languages with an editor [5]. The first family allows the end-user to draw his (or her) query without any constraint. Spatial-Query-By-Sketch [17], presented Figure 2.a, belongs to this family. The difficulty is then to generate a formal query without ambiguity (and this query must correspond to the user’s requirements). The second family guides the end-user with an editor. The end-user does not draw the query but defines the required properties (i.e., spatial relationships) and the editor proposes a visualization. Cigales [5], presented Figure 2.b, belongs to this family. In this article we are concerned with this latter approach and study the derived ambiguities. A visual ambiguity appears when several interpretations may be defined for a single visual representation. To define a new spatial relationship, the end-user points out one (or more) already available metaphor(s). A selection ambiguity appears when several metaphors may correspond to a single click.

![Image](image_url)

Figure 2 – « A road crosses a town having a forest part » expressed with (a) Spatial-Query-By-Sketch and (b) Cigales

Spatial relationships are expressed with a relative location of the metaphors (e.g., a partial overlap to express an intersection). A metaphor is represented with a graphical form (i.e., a circle-area- or a line) and an icon (i.e., to express the semantics associated with the graphical form). A query is the result of the interpretation of a visualization defined by the query editor. This visualization is built from a set of metaphors (modeling the set of geographical objects involved in the query), a set of spatial relationships (defined with the metaphors) and the properties of the language (i.e., declarative language).
2. Problems

Visual languages for geographical databases inherit from the main problem of the visual languages: the ambiguities. They appear at two levels: the building of a visualization to express a spatial configuration (building function) and the interpretation of a visualization (interpretation function) [18].

Let us consider a database with some roads, forests, towns and lakes. To manipulate this database, let us define three queries. These queries are structured as: « Show me a map such as the following properties are verified: »

- \( Q_1 \): A road crosses a town having a forest part.
- \( Q_2 \): A road borders a lake, crosses a town and a forest.
- \( Q_3 \): A road crosses the forest part of a town.

The symbolic spatial configurations are given Figure 3.

![Figure 3 – Visualization of the symbolic spatial configurations for queries Q₁, Q₂ and Q₃](image)

Within a spatial configuration several interpretations may be proposed. The visualization of query \( Q_1 \) may lead to the interpretations \( I_{11} \) and \( I_{12} \) (resp. the query \( Q_2 \) may lead to \( I_{21} \) and \( I_{22} \), the query \( Q_3 \) to \( I_3 \)). The interpretation \( I_{11} \) is the correct one for query \( Q_1 \).

- \( I_{11} \): A road crosses a town having a forest part.
- \( I_{12} \): A road crosses a town in its non-forest part
- \( I_{21} \): A road borders a lake, crosses a town and a forest
I$_{22}$: A road crosses a town, a forest and borders a lake between the two intersections
I$_{3}$: A road crosses the forest part of a town

The interpretation function may involve some visual ambiguities. The visualization of query Q$_1$ presents an ambiguity since it has at least two interpretations (I$_{11}$ and I$_{12}$). This point is very important since the number of ambiguities is a function of the number of spatial relationships involved in a query (i.e., the function is exponential). As an example, query Q$_2$ leads to 28 interpretations (for further explanations, see annexes section, part 1).

The building function may generate some selection ambiguities. To define a new spatial relationship, an end-user points out one (or more) metaphor(s) already available in the restitution space. As an example for query Q$_1$, the user may clicks on the line (modeling the road) outside the town. He or she may want to point out the entire road or the non-urban part of the road. The number of selection ambiguities follows the same evolution as the visual ambiguities do. As an example, a click on the line (modeling the road) in query Q$_2$ may correspond to 10 metaphors (for further explanations, see annexes section, part 2).

The two kinds of ambiguities (visual and selection) are due to the overlap of metaphors. A visual ambiguity is due to an ambiguity on the metaphors involved in a spatial relationship (several metaphors are available in a place modeling a spatial relationship). A selection ambiguity is due to the overlap of metaphors for a given click. The problem is that the overlap of metaphor is inevitable: it expresses the spatial relationship and defines the division into sub-objects (i.e., the results of a spatial relationship). Ambiguities cannot be suppressed.

The management of ambiguities aims at minimizing the number of ambiguities. We propose to use a placing method and an automaton for the composition of spatial relationships. The placing method respects some basic rules to insure the legibility and relies on a major paradigm: the interaction point. An interaction point models an anchor between two (or more) metaphors to express a spatial relationship. The choice of an interaction point is based on a system of weightings and priorities. Several automata (modeled with grammars) are defined. Each of them insures a specific expressive power, level of ambiguities and ease of use. The main goal is to provide an acceptable compromise between these three components.
3. Definitions

The first part introduces a formal notation to define a query. The second part classifies the various metaphors within a typology.

3.1 Formal definition of a query

A query can be modeled using several formalisms (e.g., functional, procedural). To simplify the presentation, we use here a simplified functional notation. The following principles are: an operator is identified with a label (i.e., an integer) - to be used several times in the same query -; An operator has a name and a list of arguments. A query is defined with the Query operator. In this presentation, in order to illustrate the different notions, we used two spatial operators: intersection and adjacency (noted as: ∩, ↔). To build a part of a spatial representation we use the spatial difference (noted as: Δ) and the Between operator (noted as: ↔). The spatial difference may be used to define the non-forest part of a town (i.e., difference between the spatial representations of a town and of a forest). The Between operator may be used to define a part of a linear metaphor located between two spatial relationships, e.g. a part of a road between the intersection of this road and a town and the intersection of this road and a forest. To query the database, we use the DB operator. As an example, query Q₁ is defined Figure 4.

```
Query ( 3 ∩ (1 DB (Town), 2 DB (Forest)),
        5 ∩ (1 DB (Town), 4 DB (Road)))
```

Figure 4 - Formal notation of query Q₁

To simplify the reading, the DB operator is omitted in the following. This formalism allows defining different interpretations for a query.

- I₁₁: Query (1 ∩ (Town, Forest), 2 ∩ (Town, Road))
- I₁₂: Query (3 ∩ (2 Δ(Town, 1 ∩ (Town, Forest)), Road))
- I₂₁: Query (1 ↔ (Lake, Road), 2 ∩ (Town, Road), 3 ∩ (Forest, Road))
- I₂₂: Query (4 ↔ (Lake, 3 ↔ (1 ∩ (Town, Road), 2 ∩ (Forest, Road)))

More generally : I₁₁ : Query(op₁(OG₁, OG₂), op₂(OG₁, OG₃)) ; I₁₂ : Query(op₁ (Δ(OG₁, op₂ (OG₁, OG₂)), OG₃)) ; I₂₁ : Query(op₁(OG₄, OG₁), op₂ (OG₂, OG₁), op₃ (OG₃, OG₁)) ; I₂₂ : Query(op₁(OG₄, ↔(op₂ (OG₂, OG₁), op₃ (OG₃, OG₁)))) ; I₁ : Query(op₁(OG₁, op₂(OG₂, OG₃))) where op₁ ∈ {∩, ↔, buffer, ...}, OGᵢ is a geographical object.
3.2 Typology of the various metaphors

A query is the application of several spatial operators on geographical objects. The visualization is a composition of visual overlaps on metaphors. The definition of spatial operators provides geographical objects as a result. The visual query must enlighten these new objects. We propose to classify the metaphor within two families: the original metaphors and the deducted metaphors. This last family can be refined into three sub-classes: deducted result metaphor, deducted differential one and bordered linear one.

Original metaphor: an original metaphor is a metaphor associated with a geographical entity in a query. It corresponds to a geographical object of the external database schema, e.g., a road, a town.

Deduced metaphor: a deducted metaphor is generated by the application of a spatial relationship. As an example, two deducted metaphors for query \( Q_2 \) are the forest part of the road (i.e., the part of the road that lies in the forest) and the non-forest part of the road (i.e., the part of the road outside the forest). These two metaphors are generated by the definition of the intersection between the road and the forest.

Deduced Result metaphor: a deducted result metaphor is the metaphor associated with the result of a spatial operator. Deducted result metaphors constitute a subset of deducted metaphors. As an example, a deducted result metaphor for query \( Q_2 \) is the forest part of the road (i.e., the result of the spatial intersection applied on a forest and a road, i.e., \((3 \cap \text{Forest}, \text{Road}))\).

Deduced Differential metaphor: a deducted differential metaphor is the metaphor associated with the application of a spatial difference operator. This operator is applied on a metaphor (original or deducted differential ones) and a deducted result metaphor. Deducted differential metaphors constitute a subset of deducted metaphors. As an example, a deducted differential metaphor for query \( Q_2 \) is the non-forest part of the road, i.e., \((4 \Delta \text{Road}, 3 \cap \text{Forest}, \text{Road}))\).

Bordered Linear metaphor: a bordered linear metaphor is the metaphor associated with the linear part of an object located between two deducted result metaphors. Bordered linear metaphors constitute a subset of deducted metaphors. As an example, a bordered linear metaphor for query \( Q_2 \) is the part of the road between the intersection with a
town and the intersection with a forest, i.e., \((3 \leftrightarrow (1 \cap (\text{Town, Road}), 2 \cap (\text{Forest, Road})))\).

These different metaphors in the definition of query \(Q_2\) are presented in Table 1.

<table>
<thead>
<tr>
<th>Query (Q_2)</th>
<th>Metaphors</th>
<th>Original metaphors</th>
<th>Deduced metaphors</th>
<th>Deduced Differential metaphors</th>
<th>Bordered Linear metaphors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Road</td>
<td>Road</td>
<td>Result</td>
<td>Road in forest</td>
<td>Road between the town and the forest</td>
</tr>
</tbody>
</table>

Table 1 – Illustration of the various metaphors for query \(Q_2\)

4. The visualization process

In the first part, we present the retained postulates to define the visualization process of a query. A query is either defined with a step-by-step philosophy (i.e., the addition of a new spatial relationship to the current definition of a query) or with the merge of two sub-queries (i.e., the current definition of a query is a set of independent relationships and the user defines a link between two of them). The second part analyses the different strategies to integrate a spatial relationship in a query. Last, the third part defines a placing method for the various metaphors in a query.

4.1 Postulates

Four postulates are defined in the visualization process: to favor the horizontal axis, to favor the areas, to keep the shapes and to insure a global visibility.

To favor the horizontal axis (postulate 1): the definition of a symbolic map as a query allows the logical independence between the interface level and the database level. The proposed visualization favors the user’s reading without any link to the physical relative spatial locations (which may be unknown by the user). For an occidental user, the visualization must favor the horizontal axis (from left to right in case of an oriented spatial relationship, e.g., a path from a place to another one). The second main
orientation is from up to down. The visualization of a query looking for the path from Paris to Toulouse puts Paris in the left part, Toulouse in the right part and an arrow between the two cities to identify a path (even if in the real world Toulouse is in the Southwest of Paris).

To favor the areas (postulate 2): two graphical forms, a circle and a line are proposed to visualize the graphical part of a metaphor. The use of a circle corresponds to a modeling of an object with a relevant surface (e.g., a town) - even if in the database the associated spatial representation of a town is a point -. The use of a line corresponds to a modeling of an object with a non-significant surface (e.g., a road). The complexity of a spatial query may involve a deformation of these basic graphical forms. While interpreting the query, it is more difficult to re-build a deformed circle (an area) than to follow a broken line. The method used to place the different metaphors must minimize the deformation of areas.

To keep the shapes (postulate 3): whatever the complexity of a query is, the user must be able to keep in mind the visualization of the two basic forms (circle transformed into an area and line).

To insure a global visibility (postulate 4): the user must be able to see the entire query. The breaking of the query into several sub-parts of the screen (e.g., different windows) must be avoided. A unique window must contain the whole query.

These four postulates define the set of properties the visualization process must respect. To integrate a new spatial relationship in an already defined query, several policies may be used. The following part analyses these policies and confronts them to the postulates.

4.2 Strategies

The first part defines a general strategy for a visualization. The second part proposes and studies several different strategies to integrate a new spatial relationship and determines the most favorable.
4.2.1 General strategy for a visualization

The respect of postulate 4 implies the integration of two mechanisms in a visualization strategy: the recursive division of the restitution space and the selective display.

A spatial query may be a complex composition of spatial relationships. It seems reasonable to allow the user to define independent sub-queries. The application of a spatial relationship between two components of two independent sub-queries merges them. The visualization of the different sub-queries is performed with a recursive division of the space restitution (as a quad-tree does). The user always visualizes the entire query.

Even if a spatial query involves conventional database operators, the main interest is the spatial relationships. The exhaustive visualization of the selection criteria is not realistic on a unique restitution space (obviously limited in size). The visualization strategy must concentrate on the heart of the problem: the management of spatial relationships. Secondary mechanisms can be used to provide information on the different components of a query (e.g., a click on a metaphor shows the different selection criteria).

4.2.2 Strategies of integration

Two strategies of integration of a new spatial relationship can be defined: the incremental strategy and the re-do strategy. The incremental strategy tends to keep the spatial location of the already defined metaphors. A new spatial relationship is merged with the current visualization. The metaphors may be deformed. The re-do strategy takes into account the entire set of spatial relationships to define a new location for all the metaphors. The objective is to simplify as much as possible the representation (i.e., to minimize the number of deformations).

In order to illustrate these two strategies, let us consider a complex query: « A road crosses a town having a forest part, crosses this forest and borders a lake. The lake also borders the forest ». The user can build this query step by step by adding metaphors and spatial relationship (incremental strategy). Let us suppose he or she has already defined the relationships between the road and the town, the forest and the lake (intersection between the road and the town, intersection between the road and the forest and adjacency between the road and the lake). The visualization of this step is presented Figure 5. The user may then define the intersection between the forest and the town and
finish with the adjacency between the forest ad the lake. The compared visualizations obtained with the incremental strategy and the re-do strategy are presented Figure 6.

Figure 5 - « A road crosses a town, crosses a forest and borders a lake »

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$\cap$(Forest, Town)</th>
<th>$\vartriangleleft$(Forest, Lake)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incremental</td>
<td><img src="image1" alt="Incremental Strategy" /></td>
<td><img src="image2" alt="Incremental Strategy" /></td>
</tr>
<tr>
<td>Re-do</td>
<td><img src="image3" alt="Re-do Strategy" /></td>
<td><img src="image4" alt="Re-do Strategy" /></td>
</tr>
</tbody>
</table>

Figure 6 – Comparison of the incremental and the re-do strategy

Three criteria allow to make a choice between these two strategies: the visualization process must allow a declarative language, the areas must be favored (postulate 2), and the legibility.

A visual language dedicated to an end-user must be declarative. The incremental strategy favors the oldest metaphors. It minimizes the displacement of already defined objects. The re-do strategy equally treats all metaphors (whatever the date of creation). So the order of definition is important in the first strategy. The second strategy insures stability in the structure of the representation. The first one is close to a procedural approach. In the case of a declarative language, the second approach seems to be better. Furthermore, postulate 2 favors the priority to the areas. The incremental strategy cannot integrate a system of priority for this kind of metaphor. This system may be in conflict with the system based on the date of creation. The second strategy can integrate, at each new step of visualization, a system of priority for the set of areas involved in a query.

Finally, postulate 3 requires keeping the shapes. The incremental strategy can quickly lead to modify the shape of the metaphors. To keep the initial configuration while introducing a new spatial relationship, may lead to very complex shapes (and even some undesired spatial relationships - e.g., visual ambiguity).
The re-do strategy may displace the metaphors at each step of visualization. The user may be upset by these changes. In fact, only the re-do strategy is able to respond to the different constraints we defined: declarativity, priority of areas and visibility. We choose this strategy during the development of our placing method.

4.3 Placing method

The placing method aims at defining the relative locations of the different metaphors. This placement describes the required spatial relationships defined in the query. The objective is to minimize the number of ambiguities. The basic paradigm is the interaction point. The first part defines this notion and presents the mechanisms used to anchor the metaphors. Postulate 2 requires favoring the areas. We introduce in the second part the notion of grape and study the implications of this notion on the placing method. Last, the visualization requires the definition of a dynamic mechanism to handle the metaphors. This mechanism is presented in the third part.

4.3.1 Interaction point

We first define the interaction point. The second part illustrates how these points are used to express the spatial relationships. The objective is to maximize the weighting, presented in the third part and the priority, presented in the fourth part.

4.3.1.1 Definition

An interaction point is defined on a metaphor to anchor another metaphor. Physically, this point is located on a graphical primitive of a metaphor.

By default, original metaphors of type « area » are represented with a circle and have eight interaction points (Figure 7.a). Original metaphors of type « line » are represented with a segment and have five interaction points (Figure 7.b).

![Figure 7 - Interaction points on basic graphical forms](image-url)
4.3.1.2 Support of spatial relationships

The expression of a spatial relationship is performed by the overlap of one (e.g., for the adjacency) or two (e.g., for the intersection) interaction points. Table 2 presents the anchor of two spatial relationships for the different metaphors.

| Spatial relationship | Metaphor | | |
|---------------------|---------| | |
| Intersection        | Area    | | |
|                      | Line    | | |
| Adjacency           | Area    | | |
|                      | Line    | | |

Table 2 - Examples of expression of a spatial relationship with its anchors

Interaction points should not be chosen by chance. We must consider (1) the already available metaphors and spatial operators defined in the query and (2) the postulates. To do so, a system of weighting is defined.

4.3.1.3 System of weighting

The objective of the system of weighting is to limit the ambiguities (e.g., the expression of spatial relationships that have not been explicitly required). A level of weighting is given to each interaction point for each metaphor (area or line) and for each type of spatial relationship (e.g., intersection, adjacency). We define five levels of weighting (from the minimal one to the maximal one): non-relevant, occupied, inhibited, problematic and free. Postulate 1 (to favor the horizontal axis and then the vertical one) implies that the diagonals are not used. For the representation of an « area » metaphor, the choice of the point Northeast (NE) to anchor another « area » to express an adjacency is not relevant. Whenever a point materializes a spatial relationship, it is classified as « occupied ». Figure 8 presents a metaphor of type « area » named A with the points « Northeast » and « Southeast » as occupied. The expression of a spatial relationship may involve some interaction points to be inaccessible. They are said « inhibited ». Let us consider the intersection of two areas (Figure 8). The point « East » of the area A is included in the spatial representation of the area B. An expression of a spatial relationship of A with this point introduces an

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3 The weighting « non-relevant » is noted « - ». 
interaction with area B. This point is therefore classified as inhibited even if it does not support any spatial relationship. Furthermore, the expression of a spatial relationship may render difficult an access to some interaction points. Figure 8 illustrates such a configuration. The placement of a line in adjacency with Area A on the point « North » induces an adjacency with Area B. To avoid such a problem, the line must be broken (e.g., point South). The interaction point is classified as « problematic ». An interaction point classified as « free » can be chosen to express a spatial relationship without any difficulty.

![Figure 8 – Intersection of two metaphors « area »](image)

When a metaphor is created, an array of initial weightings is provided (Table 3 for an area). The interaction points are either free or non-relevant.

<table>
<thead>
<tr>
<th>Interaction point</th>
<th>⨅Area</th>
<th>⨅Line</th>
<th>≪Area</th>
<th>≪Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>-</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>NE</td>
<td>Free</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>E</td>
<td>Free</td>
<td>-</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>SE</td>
<td>Free</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
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<td>-</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>SW</td>
<td>Free</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>W</td>
<td>-</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>NW</td>
<td>Free</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3 - Initial weighting of a metaphor « area »

During the placing of other metaphors, this array is modified. Let us consider the visualization of query \( Q_1 \) (Figure 3). A first step of placing may be to locate the forest on the eastern side of the town. The weightings are presented Table 4.
To anchor a road on a town, we must maximize the weightings of a couple of opposite points in the column associated with the intersection with a line. The couple North-South is chosen.

The deducted metaphors may also support some spatial relationships. They have also a weighting. Table 5 presents the weightings of the forest part of a town for query Q₁ (deduced result metaphor of the intersection between a town and a forest).

<table>
<thead>
<tr>
<th>Interaction point</th>
<th>⌈Area⌉</th>
<th>⌈Line⌉</th>
<th>⌈↓Area⌉</th>
<th>⌈↓Line⌉</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
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<td>Inhibited</td>
<td>Inhibited</td>
<td>Inhibited</td>
</tr>
<tr>
<td>SE</td>
<td>Occupied</td>
<td>Occupied</td>
<td>Occupied</td>
<td>Occupied</td>
</tr>
<tr>
<td>S</td>
<td>-</td>
<td>Free</td>
<td>Free</td>
<td>Problematic</td>
</tr>
<tr>
<td>SW</td>
<td>Free</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>W</td>
<td>-</td>
<td>Problematic</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>NW</td>
<td>Free</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N</td>
<td>-</td>
<td>Free</td>
<td>Free</td>
<td>Problematic</td>
</tr>
<tr>
<td>NE</td>
<td>Occupied</td>
<td>Occupied</td>
<td>Occupied</td>
<td>Occupied</td>
</tr>
</tbody>
</table>

Table 4 – Weightings associated with a town after the intersection with a forest - Query Q₁

Whenever the maximization of the weightings cannot provide a unique solution, a priority mechanism is performed.

4.3.1.4 System of priority

The priority mechanism aims at respecting postulate 1 (to favor the horizontal axis then the vertical one). A level of priority is defined for each type of metaphors (line, area) and for each spatial relationship (e.g., intersection, adjacency). A high priority is expressed with a high number. Table 6 presents the levels of priority for an area. To intersect an area, two interaction points and an intermediate point are considered.
Let us consider the visualization of query $Q_1$ (Figure 3). The relative location of the forest versus the town cannot only be defined with the system of weighting. The town has not yet any spatial relationship. The system of weighting proposes four pairs of points to support an intersection with a forest (Northeast and Southeast, Southeast and Southwest, Southwest and Northwest, Northwest and Northeast). The system of priority allows choosing a unique pair: Southeast and Northeast (priority 8: $4 + 0 + 4$). The forest is therefore located on the eastern part of the town.

### 4.3.2 Grape

Postulate 2 states that it is necessary to minimize the deformations of areas. We propose to place in priority the areas and then to look at the placement of lines. The visualization of a query is therefore divided into two parts. First, the placements of the areas are evaluated and then the drawing of the lines within this set of areas is performed. For each of these steps, the placing method with the interaction points is performed\(^4\). We introduce the concept of grape to favor the placements of areas and we study the properties of the grapes (to evaluate the impacts upon the placing method).

**Graphical Link:** A graphical link describes the existence of a spatial relationship between metaphors of type « area ».

As an example, in query $Q_1$, it exists a graphical link between the town and the forest. Graphical links are modeled with a dependence graph $G (S, A, \Psi, \alpha, \beta)$ where $S$ is the set of nodes modeling areas, $A$ is the set of edges modeling a graphical link defined by the incident function $\Psi$: $A \rightarrow S \times S$, $\alpha$ and $\beta$ are applications from $A$ to $S$ mapping to each edge a its origin node $\alpha(a)$ and destination node $\beta(a)$.

---

\(^4\) The rebuilding of the whole line is insured by a mechanism of vector association (based on scalar products and vector analysis) not developed in this article.
**Grape**: A grape is a connected component of $G$.

In other words, a grape is a set of areas linked by a graphical link. The set \{town, forest\} is a grape for query $Q_1$.

The visualization process applies the placing method on each grape of a query (each of these grapes is independent from the other ones). Whenever the grapes are linked with lines, the drawing is based on a calculus of scalar vectors and vector products (not developed here). Lines may not link grapes since the logical link between two grapes may be an alphanumeric selection criterion instead of being a spatial relationship.

**Path**: a path, with a length $q > 0$ is a sequence of edges $\mu = (a_1, a_2, ..., a_q)$ such as each edge (except $a_1$ and $a_q$) of the sequence has its origin node equal to the destination node of the previous edge and its destination node equal to the origin node of the following edge:

$$\text{if } 0 < i < q: \beta(a_i) = \alpha(a_{i+1})$$

**Cycle**: a cycle is a path $C = (a_1, a_2, ..., a_q)$ such as $\beta(a_q) = \alpha(a_1)$

Some metaphors cannot be modeled with circles. A grape with a cycle requires the deformation of an area of the cycle. To visualize a grape in which several spatial relationships are defined on a single area may be impossible. The case is similar if the query requires to express a spatial relationship on a deducted result metaphor (Table 7: the weightings of the forest part of a town shows that it is impossible to express an intersection of this metaphor with an other metaphor of type « area »).

### 4.3.3 Dynamic treatment

The deformation of a metaphor must respect postulate 3 (evident graphical link between the metaphors of the two families - circle, line). We favor the use of arcs of a circle for the visualization of areas. The lines are represented by connected parallel or perpendicular segments to make a clear difference between areas and lines.

We distinguish two kinds of deformation: the deformation by extension and the deformation by composition. The extension allows increasing the number of interaction points on a graphical object by stretching it. For an area, a pair of interaction points (North-South or East-West) is replaced by a pair of segments (each of them has five interaction points), for a line, parallel segments are juxtaposed. The first column of
Table 7 presents an example of extension of areas and lines. The composition allows increasing the number of interaction point by changing the form of the metaphors. For an area, a set of arcs of a circle and segments are juxtaposed. The composition upon areas is mandatory whenever a cycle appears in a grape. The last column of Table 7, first line, the visualization of a cycle (made with three areas in intersection) is proposed. For a line, the composition is performed by the use of parallel and perpendicular segments (Table 7 last column, last line).

<table>
<thead>
<tr>
<th>Extension</th>
<th>Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image1" alt="Image" /></td>
</tr>
<tr>
<td>Line</td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image3" alt="Image" /></td>
</tr>
</tbody>
</table>

Table 7 – Examples of deformations for a metaphor

The policy of placing aims at reducing the ambiguities. It cannot suppress them. Beyond this policy, a grammar modeling the interactions between an end-user and a query editor is proposed.

5. **Grammars of interaction**

A visual language for an end-user is a compromise between the expressive power and the ease of manipulation. Defining a simple visual language able to express very complex queries is not realistic. A grammar is a well-adapted concept to study the expressive power of a language. It allows modeling the authorized interactions, the accessible metaphors and the way to combine them. We define in the following a set of grammar and study the links between the different levels of ambiguities.

The first part models a query with a graph and defines the associated functions. These functions are used in the second part that presents the vocabulary associated with the grammars. The third part compares the different grammars and their expressive power. The fourth part studies the links with the different levels of ambiguities. The last part is a synthesis on the appropriateness of the different grammars and the different categories of end-users.

5.1 **Modeling of a spatial query**

A spatial query is modeled with an oriented graph $Q (S, A, \alpha, \beta, \nu, \varepsilon)$ where:
• S is a set of nodes (i.e., the original metaphors and the spatial relationships defined in a query): \( S = \{s_1, s_2, \ldots, s_p\} \);

• A is a set of edges, disjoint from S: \( A = \{a_1, a_2, \ldots, a_q\} \);

• \( \alpha \) and \( \beta \) have the same definitions as previous;

• \( \nu \) is a labeling function for the nodes:

\[
\nu : S \rightarrow \{GF \cup SR\}
\]

where GF is the set of graphical forms that can be employed by the end-user:

\[
GF = \{\text{Area, Line}\}
\]

where SR is the set of spatial relationships, i.e., in this presentation:

\[
SR = \{\cap, \prec, \Delta, \leftrightarrow\}
\]

• \( \varepsilon \) is a labeling function. This function allows defining an order between the metaphors involved in a spatial relationship since spatial operators may be non-symmetric (e.g. difference): \( \varepsilon : A \rightarrow \text{IN}^* \)

To simplify a query \( Q(S, A, \alpha, \beta, \nu, \varepsilon) \) is noted \( Q \) whenever no ambiguity appears.

Direct link: \( s_i (s_i \in S) \) is a successor of \( s (s \in S) \) in \( Q \) IFF it exists an edge \( (s, s_i) \).

The set of successors for a node \( s \) is noted \( \Gamma^+_{R}(s) \).

\[
\Gamma^+_{R}: S \rightarrow S^p
\]

\[
\Gamma^+_{R}(s) = \{s_i \in S / \exists a \in A, \alpha(a) = s \land \beta(a) = s_i\}
\]

The \( i^{th} \) successor of a node \( s \), in the order defined by \( \varepsilon \) on the edges leaving \( s \) is noted \( \Gamma^+_{R_i}(s) \).

\[
\Gamma^+_{R_i}: S \rightarrow S
\]

\[
\Gamma^+_{R_i}(s) = \{s_2 \in S / \exists a \in A, \alpha(a) = s \land \beta(a) = s_2 \land \varepsilon(a) = i\}
\]

Depth

The depth of a graphical form is noted D:

\[
S \rightarrow \text{IN}^*
\]

\[
\nu(s) \in GF \Rightarrow D(s) = 1
\]

\[
\nu(s) \in SR \Rightarrow D(s) = \sum_{i=1}^{\text{Card}(\Gamma^+_{R_i}(s))} \Gamma^+_{R_i}(s)
\]

The depth of a graphical form is used during the selection process. It allows pointing out a unique metaphor from a set of candidates (based on the maximization of the depth). Let us consider query \( Q_1 \) (Figure 3) with interpretation \( I_{11} \). Let us consider that
the user clicks on the town, outside all spatial relationships, and that the user cannot access original metaphors. Two metaphors correspond to this selection. Figure 9 presents these metaphors with an algebraic tree. For each node, the depth of the metaphor is given. It is possible to consider that the user wishes to select the metaphor without any spatial relationship (e.g., the non-forest part of the town that is not crossed by the road). This metaphor has the maximum depth. Let us now consider query Q₃ and that the user clicks on the part of the road that is inside the forest and the town and that he or she cannot access deducted differential metaphors. Five metaphors may correspond to this selection (Figure 10). It is possible to consider that he or she wants to point out the closest metaphor around his or her selection. This metaphor is the part of the road that is inside the forest and the town. This metaphor has the maximum depth.

\[\text{Figure 9 – Metaphors for a click on the town in query Q₁ and the associated depths}\]

\[\text{Figure 10 – Metaphors for a click on the road in the town and in the forest in query Q₃ and the associated depths}\]

### 5.2 Vocabulary

The vocabulary of the grammars is defined with three main modes derived from the classification of the metaphors. The first one is the Global mode (G). The second one is the Partial mode (P). The third one is the Between mode (B).

The Global mode allows pointing out an original metaphor. The partial mode allows pointing out a deducted result metaphor or a deducted differential metaphor. The Between mode allows pointing out a bordered linear metaphor. The Partial mode is divided into two modes: P_{DR} and P_{DD}. P_{DR} allows accessing the deducted result metaphor with the maximal depth. The P_{DD} mode allows accessing the differential
deduced metaphor with the maximal depth. The vocabulary of interaction is \( V = \{G, P, P_{DR}, P_{DD}, B\} \).

5.3 Grammars and expressive power

The first interaction is obviously in the Global mode since there is no spatial relationship and therefore no deduced metaphor. From this definition and due to the incompatibilities, seventeen grammars may be defined from the vocabulary. For each of them, we present the expressive power and illustrate it with queries \( Q_1, Q_2 \) and \( Q_3 \). These grammars are classified into three parts: grammars without a change of mode, grammars with an implicit change of mode, grammars with an explicit change of mode.

5.3.1 Grammars without a change of mode

Grammar \( G^* \) limits the definition of spatial relationships to original metaphors. This grammar corresponds to a query language based on predicates. It is not worth calculating the results associated with the various spatial relationships. Among the five interpretations for queries \( Q_1, Q_2 \) and \( Q_3 \), only \( I_{11} \) and \( I_{21} \) can be expressed with this grammar. This expressive power is currently used in visual languages for geographical databases ([27]).

Grammar \( P^* \) allows combining spatial operators on deduced result metaphors or deduced differential metaphor. It is not possible to define a spatial relationship on original metaphors (except the two first defined in a grape). Interpretations \( I_{12} \) and \( I_3 \) can be expressed.

Grammar \( P_{DR}^* \) allows combining spatial relationships on deduced result metaphors with the maximal depth. Interpretation \( I_3 \) can be expressed.

Grammar \( P_{DD}^* \) allows combining spatial relationships on deduced differential metaphors with the maximal depth. Interpretation \( I_{12} \) can be expressed.

5.3.2 Grammars with an implicit change of mode

Grammar \( GB^* \) allows, after having defined a second relationship with the Global mode, expressing new relationships (among the already available). The management of linear metaphors is only concerned with this mode. Interpretation \( I_{22} \) can be expressed.

Grammar \( G(G|B)^* \) allows an access to original metaphors and to bordered linear metaphors. No deduced metaphor can be used on a metaphor modeled with an area. This language is therefore equivalent to \( G^* \) for the areas. On linear metaphors, a spatial
relationship can be defined without taking into account the other ones (mode G) or between two of them (mode B). Interpretations $I_{11}$, $I_{21}$ and $I_{22}$ can be expressed.

5.3.3 Grammar with an explicit change of mode

Grammar $(G|P)^*$ allows an access to all metaphors except deduced bordered linear metaphors. The expressive power is very high. Interpretations $I_{11}$, $I_{12}$, $I_{21}$ and $I_3$ can be expressed.

Grammar $(G|P_{DR})^*$ allows an access to original metaphors and deduced result metaphors with the maximal depth. This grammar is used in some visual languages able to manage the result of an operator [29]. Interpretations $I_{11}$, $I_{21}$ and $I_3$ can be expressed.

Grammar $(G|P_{DD})^*$ allows an access to original metaphors and to deduced differential metaphors with the maximal depth. Interpretations $I_{11}$, $I_{12}$ and $I_{21}$ can be expressed.

Grammar $(P|B)^*$ forbids the composition of spatial relationships on original metaphors. Interpretations $I_{12}$, $I_{22}$ and $I_3$ can be expressed.

Grammar $(P_{DR}|P_{DD})^*$ allows an access to spatial relationships on deduced result metaphors or on deduced differential ones with the maximal depth. Interpretations $I_{12}$ and $I_3$ can be expressed.

Grammar $(P_{DD}|B)^*$ allows an access to spatial relationships on deduced differential metaphors with the maximal depth or on deduced bordered linear metaphors. Interpretations $I_{12}$ and $I_{22}$ can be expressed.

Grammar $(G|P_{DR}|P_{DD})^*$ allows an access to spatial relationships on original metaphors and on deduced result metaphors or on deduced differential metaphors with the maximal depth. Interpretations $I_{11}$, $I_{12}$, $I_{21}$ and $I_3$ can be expressed.

Grammar $(G|P_{DR}|B)^*$ allows an access to spatial relationships on the original metaphors, on deduced result metaphors with the maximal depth and on deduced bordered linear metaphors. Interpretations $I_{11}$, $I_{21}$, $I_{22}$ and $I_3$ can be expressed.

Grammar $(G|P_{DD}|B)^*$ allows an access to spatial relationships on original metaphors, deduced differential metaphors with the maximal depth and on deduced bordered linear metaphors. Interpretations $I_{11}$, $I_{12}$, $I_{21}$ and $I_{22}$ can be expressed.

Grammar $(G|P|B)^*$ allows an access to all metaphors. All interpretations given as an example can be expressed.

Grammar $(G|P_{DR}|P_{DD}|B)^*$ allows an access to all metaphors. The depth mechanism makes a selection between all candidates. All interpretations given as an example can be expressed.
Table 8 sums up the different grammars and the associated expressive power based on the given interpretations.

<table>
<thead>
<tr>
<th>Grammar</th>
<th>$I_{11}$</th>
<th>$I_{12}$</th>
<th>$I_{21}$</th>
<th>$I_{22}$</th>
<th>$I_{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G^*$</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P^*$</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{DR}^*$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>$P_{DD}^*$</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$GB^*$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>$(G</td>
<td>B)^*$</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>$(G</td>
<td>P)^*$</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>$(G</td>
<td>P_{DR}^*)$</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>$(G</td>
<td>P_{DD})^*$</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>$(P</td>
<td>B)^*$</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>$(P_{DR}</td>
<td>P_{DD})^*$</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(P_{DD}</td>
<td>B)^*$</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(G</td>
<td>P_{DR}</td>
<td>P_{DD})^*$</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>$(G</td>
<td>P_{DR}</td>
<td>B)^*$</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>$(G</td>
<td>P_{DD}</td>
<td>B)^*$</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>$(G</td>
<td>P</td>
<td>B)^*$</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>$(G</td>
<td>P_{DR}</td>
<td>P_{DD}</td>
<td>B)^*$</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Table 8 – Grammars and expressive power

### 5.4 Grammars and ambiguities

Table 9 compares the different grammars and the different levels of ambiguities (visual or selection). The levels of ambiguities are illustrated with query $Q_2$. The first column defines the grammars. The second one gives the number of possible interpretations (producing a visual ambiguity) for query $Q_2$. « - » means that no ambiguity appears. The third column generalizes the notion of visual ambiguity whatever the query is. The fourth column indicates the maximum number of metaphors as a candidate one for a selection in query $Q_2$. « - » means that no selection ambiguity appears. The fifth column generalizes the notion of selection ambiguity whatever the query is.
Table 9 – Grammars and ambiguities

<table>
<thead>
<tr>
<th></th>
<th>Visual ambiguity for $Q_2$</th>
<th>General visual ambiguity</th>
<th>Selection ambiguity for $Q_2$</th>
<th>General selection ambiguity</th>
</tr>
</thead>
<tbody>
<tr>
<td>B*</td>
<td>-</td>
<td>Yes</td>
<td>2</td>
<td>Yes</td>
</tr>
<tr>
<td>G*</td>
<td>-</td>
<td>No</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>$P_{DR}$*</td>
<td>-</td>
<td>No</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>G(G</td>
<td>B)*</td>
<td>2</td>
<td>Yes</td>
<td>3</td>
</tr>
<tr>
<td>(G</td>
<td>P_{DR})*</td>
<td>-</td>
<td>No</td>
<td>-</td>
</tr>
<tr>
<td>(G</td>
<td>P_{DR}</td>
<td>B)*</td>
<td>2</td>
<td>Yes</td>
</tr>
<tr>
<td>$P_{DD}$*</td>
<td>6</td>
<td>Yes</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>(P_{DD}</td>
<td>P_{DD})*</td>
<td>6</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>(P_{DD}</td>
<td>B)*</td>
<td>8</td>
<td>Yes</td>
<td>3</td>
</tr>
<tr>
<td>(G</td>
<td>P_{DD})*</td>
<td>12</td>
<td>Yes</td>
<td>2</td>
</tr>
<tr>
<td>(G</td>
<td>P_{DD}</td>
<td>P_{DD})*</td>
<td>12</td>
<td>Yes</td>
</tr>
<tr>
<td>(G</td>
<td>P_{DD}</td>
<td>B)*</td>
<td>15</td>
<td>Yes</td>
</tr>
<tr>
<td>P*</td>
<td>15</td>
<td>Yes</td>
<td>7</td>
<td>Yes</td>
</tr>
<tr>
<td>(G</td>
<td>P_{DD}</td>
<td>P_{DD}</td>
<td>B)*</td>
<td>15</td>
</tr>
<tr>
<td>(P</td>
<td>B)*</td>
<td>17</td>
<td>Yes</td>
<td>9</td>
</tr>
<tr>
<td>(G</td>
<td>P)*</td>
<td>25</td>
<td>Yes</td>
<td>8</td>
</tr>
<tr>
<td>(G</td>
<td>P</td>
<td>B)*</td>
<td>28</td>
<td>Yes</td>
</tr>
</tbody>
</table>

In the following, we study more in detail two grammars: $(G|P_{DR})*$ and $(G|P|B)*$. The first one is regularly used in visual languages for GIS and the second one provides a maximal expressive power.

Grammar $(G|P_{DR})*$ allows suppressing all visual ambiguities and all selection ambiguities. A spatial relationship deals with the deducted result metaphor with the maximal depth as soon as this relationship is defined in a place where at least another spatial relationship is already represented. Outside an already defined spatial relationship, the new one concerns the original metaphor. No visual ambiguity may appear. A selection where a spatial relationship is expressed concerns the deducted result metaphor with the maximal depth. Outside all spatial relationships, the new one concerns the original metaphor. No selection ambiguity may appear. The interaction between the end-user and the query editor is simple. The basic action is a selection of a metaphor in the restitution area and the definition of a new spatial relationship. The system is able to detect the relevant metaphors.

Grammar $(G|P|B)*$ presents the higher level of visual ambiguities and the higher level of selection ambiguities. All metaphors are accessible. The overlap of metaphors is therefore very important. The interaction between the end-user and the query editor is
obviously more complex. A dialogue must be introduced as soon as a selection ambiguity appears.

5.5 Synthesis

Each of the grammars allows a specific expressive power. They define a level of ambiguities and a level of interaction with an end-user. The more the grammar authorizes an access to a high number of metaphors, the highest the expressive power is. In correlation, this power increases the level of ambiguities and the interactions with the end-user.

A simple grammar, with an acceptable expressive power, with a limited level of ambiguities (e.g., \((G|P_{DR})^*\)) is convenient for an occasional end-user. Generally, the queries are simple. In opposition with an intensive professional user, the expressive power must be higher. Nevertheless, this kind of users may be able to tackle the subtleties of the query language. These difficulties are obviously less problematic for this kind of users than with an occasional end-user.

6. Conclusion

Geographic applications are more and more used in the economy. The development of visual languages may spread the applications to end-users that are not familiar with a computer. Queries should not be pre-defined. An interaction to express free queries must be provided. Within the visual languages for GIS we define two families: the languages based on a query editor and the languages based on a drawing tool. Our study concerns the first family.

In this article, we present the study of the problems involved by a visual representation of spatial relationships as a query language. Two main problems appear, namely the visual ambiguities and the selection ambiguities. To reduce these drawbacks we couple a drawing process with grammars of interaction. The drawing process defines the rules to visualize a query and a placing method based on interaction points. A grammar of interaction models the links between an end-user and the query editor. Each defined grammar offers a specific expressive power. The levels of ambiguities and the complexity of interactions are closely linked to the expressive power of the retained grammar.

The more the grammar is powerful, the more the level of ambiguity may be important and therefore the more the link with the end-user is complex. The problem is
then to adjust the relevant grammar to an end-user. The aim is not to provide the maximum of the expressive power whatever is the end-user but to fit it. From this formal study, several concrete interfaces may be developed depending on the desired category of end-users.

7. References


8. Annexes

8.1 28 interpretations for the visualization of query Q2

The Figure 11 shows all the possible combinations to define a query which may have the same visualization as query Q2.

<table>
<thead>
<tr>
<th>Part of the road considered for the spatial operator</th>
<th>Permutation 1</th>
<th>Permutation 2</th>
<th>Permutation 3</th>
<th>Permutation 4</th>
<th>Permutation 5</th>
<th>Permutation 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>The entire road</td>
<td>∩1 \text{∩} ∩2</td>
<td>P1</td>
<td>P1</td>
<td>P1</td>
<td>P1</td>
<td>P1</td>
</tr>
<tr>
<td>Its non urban part</td>
<td></td>
<td>P1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The part not bordered by the lake</td>
<td></td>
<td></td>
<td>P2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Its non-forest part</td>
<td></td>
<td></td>
<td></td>
<td>P3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part outside of the town and not bordered by the lake</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>P4</td>
<td></td>
</tr>
<tr>
<td>Its non urban and non-forest part</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>P5</td>
</tr>
<tr>
<td>Part outside of the forest and not bordered by the lake</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Between the town and the forest</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

28

∩1 stands for the definition of the intersection between the road (or a part of the road) and the town.

∩2 stands for the definition of the intersection between the road (or a part of the road) and the forest.

\text{∩∩} stands for the definition of the adjacency between the road (or a part of the road) and the lake.

P_i stands for a query already considered in permutation i

Figure 11 - Combinations of spatial operators leading to the query Q2 visualization
The corresponding queries are:

“Show me a map such as the following properties are verified:”

<table>
<thead>
<tr>
<th>Permutation</th>
<th>A road crosses a town, borders a lake and crosses a forest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A road crosses a town, borders a lake and crosses a forest in its non urban part</td>
</tr>
<tr>
<td></td>
<td>A road crosses a town, borders a lake and crosses a forest in the part not bordered by the lake</td>
</tr>
<tr>
<td></td>
<td>A road crosses a town, borders a lake and crosses a forest in the part outside of the town and not bordered by the lake</td>
</tr>
<tr>
<td></td>
<td>A road crosses a town, borders a lake in its non urban part and crosses a forest in its non urban part</td>
</tr>
<tr>
<td></td>
<td>A road crosses a town, borders a lake in its non urban part and crosses a forest in the part not bordered by the lake</td>
</tr>
<tr>
<td></td>
<td>A road crosses a town, borders a lake in its non urban part and crosses a forest in the part outside of the town and not bordered by the lake</td>
</tr>
<tr>
<td>2</td>
<td>A road crosses a town, crosses a forest and borders a lake in its non-forest part</td>
</tr>
<tr>
<td></td>
<td>A road crosses a town, crosses a forest and borders a lake in its non urban and non-forest part</td>
</tr>
<tr>
<td></td>
<td>A road crosses a town, crosses a forest and borders a lake between the town and the forest</td>
</tr>
<tr>
<td></td>
<td>A road crosses a town, crosses a forest in its non urban part and borders a lake in its non-forest part</td>
</tr>
<tr>
<td></td>
<td>A road crosses a town, crosses a forest in its non urban part and borders a lake in its non urban and non-forest part</td>
</tr>
<tr>
<td></td>
<td>A road crosses a town, crosses a forest in its non urban part and borders a lake between the town and the forest</td>
</tr>
<tr>
<td>3</td>
<td>A road borders a lake, crosses a town in the part not bordered by the lake and crosses a forest</td>
</tr>
<tr>
<td></td>
<td>A road borders a lake, crosses a town in the part not bordered by the lake and crosses a forest in its non urban part</td>
</tr>
<tr>
<td></td>
<td>A road borders a lake, crosses a town in the part not bordered by the lake</td>
</tr>
</tbody>
</table>
and crosses a forest in the part not bordered by the lake
A road borders a lake, crosses a town in the part not bordered by the lake and crosses a forest in the part outside of the forest and not bordered by the lake

| Permutation | A road borders a lake, crosses a forest and crosses a town in its non-forest part
A road borders a lake, crosses a forest and crosses a town in the part outside of the forest and not bordered by the lake
A road borders a lake, crosses a forest in the part not bordered by the lake and crosses a town in its non-forest part
A road borders a lake, crosses a forest in the part not bordered by the lake and crosses a town in the part outside of the forest and not bordered by the lake
A road crosses a forest, crosses a town in its non-forest part and borders a lake in its non urban part
A road crosses a forest, crosses a town in its non-forest part and borders a lake in its non-forest part
A road crosses a forest, crosses a town in its non-forest part and borders a lake in its non urban and non-forest part
A road crosses a forest, crosses a town in its non-forest part and borders a lake between the town and the forest
A road crosses a forest, borders a lake in its non-forest part and crosses a town in the part not bordered by the lake
A road crosses a forest, borders a lake in its non-forest part and crosses a town in the part outside of the forest and not bordered by the lake |

8.2 10 metaphors for a click on the line (modeling the road) in query $Q_2$

The Figure 12 shows all the parts of the road existing in query $Q_2$. Let us consider that the user clicks on the part of the road between the town and the lake (visualized by an arrow on Figure XY). This figure shows 10 corresponding metaphors.
Figure 12 - Metaphors corresponding to a click on the road in query $Q_2$

- **O**: Road
  - **DD$_1$**: $\Delta$($Road$, $\cap$ ($Road$, $Town$))
    - Its non-urban part
  - **DD$_2$**: $\Delta$($Road$, $\Leftrightarrow$ ($Road$, $Lake$))
    - The part not bordered by the lake
  - **DD$_3$**: $\Delta$($Road$, $\cap$ ($Road$, $Forest$))
    - Its non-forest part
  - **DD$_4$**: $\Delta$($\Delta$($Road$, $\cap$ ($Road$, $Town$)), $\Leftrightarrow$ ($Road$, $Lake$))
    - Part outside of the town and not bordered by the lake
  - **DD$_5$**: $\Delta$($\Delta$($Road$, $\cap$ ($Road$, $Town$)), $\cap$ ($Road$, $Forest$))
    - Its non-urban and non-forest part
  - **DD$_6$**: $\Delta$($\Delta$($Road$, $\Leftrightarrow$ ($Road$, $Lake$)), $\cap$ ($Road$, $Forest$))
    - Part outside of the forest and not bordered by the lake
  - **DD$_7$**: $\Delta$($\Delta$($Road$, $\Leftrightarrow$ ($Road$, $Lake$)), $\cap$ ($Road$, $Forest$), $\Leftrightarrow$ ($Road$, $Lake$))
    - Part outside of the forest, outside of the town and not bordered by the lake

- **BL$_1$**: $\leftrightarrow$($\cap$ ($Road$, $Town$), $\cap$ ($Road$, $Forest$), $Road$)
  - Between the town and the forest

- **BL$_2$**: $\leftrightarrow$($\cap$ ($Road$, $Town$), $\Leftrightarrow$ ($Road$, $Lake$), $Road$)
  - Between the town and the lake